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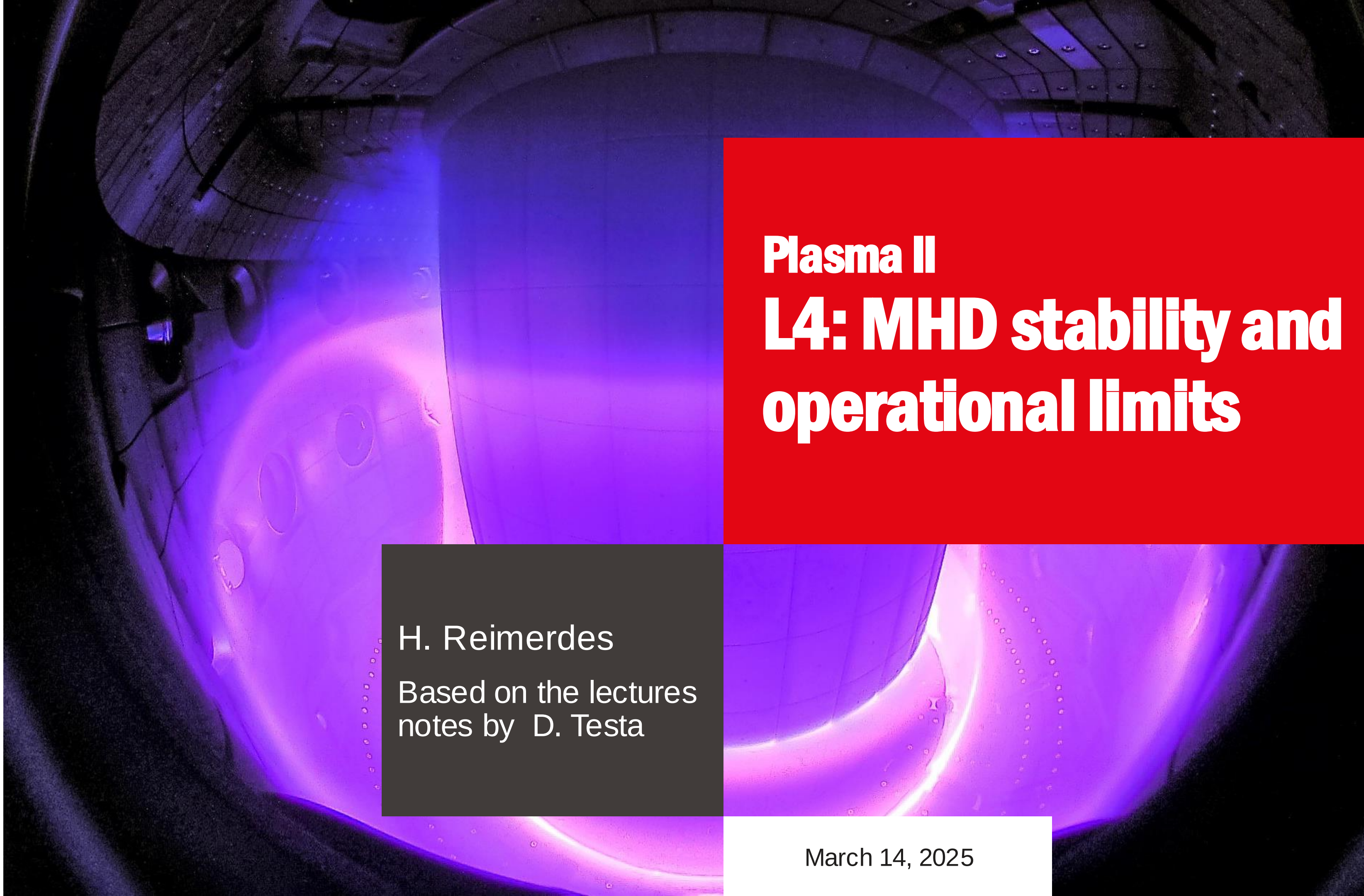

Plasma II
L4: MHD stability and operational limits

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Based on the lectures notes by D. Testa

March 14, 2025

Swiss Plasma Center

MHD lectures

- L2: The Magnetohydrodynamic (MHD) description of a plasma
- L3: MHD equilibrium configurations of interest for magnetic confinement fusion
- L4: MHD stability and operational limits

Outline

- MHD stability of the tokamak configuration
  - Conceptual examples of instabilities
  - Linear stability analysis
- Operational limits in tokamak plasmas

Material

- See also EPFL MOOC “Plasma physics: Introduction”, module #3g (#3h), “Plasma physics: Applications”, #7c
  - https://learning.edx.org/course/course-v1:EPFLx+PlasmaIntroductionX+3T2016/home
  - https://learning.edx.org/course/course-v1:EPFLx+PlasmaApplicationX+3T2016/home
- Wesson, *Tokamaks* - Third Edition, Ch. 6.1-6.7, 7.1-7.3, 7.7-7.9, 7.18
- Zohm, *MHD Stability of Tokamaks*, Wiley-VCH, Ch. 3.1-3.3, 4.1-4.2

Stability of the MHD equilibrium

Def.: MHD equilibrium ≡ Sum of all forces is zero

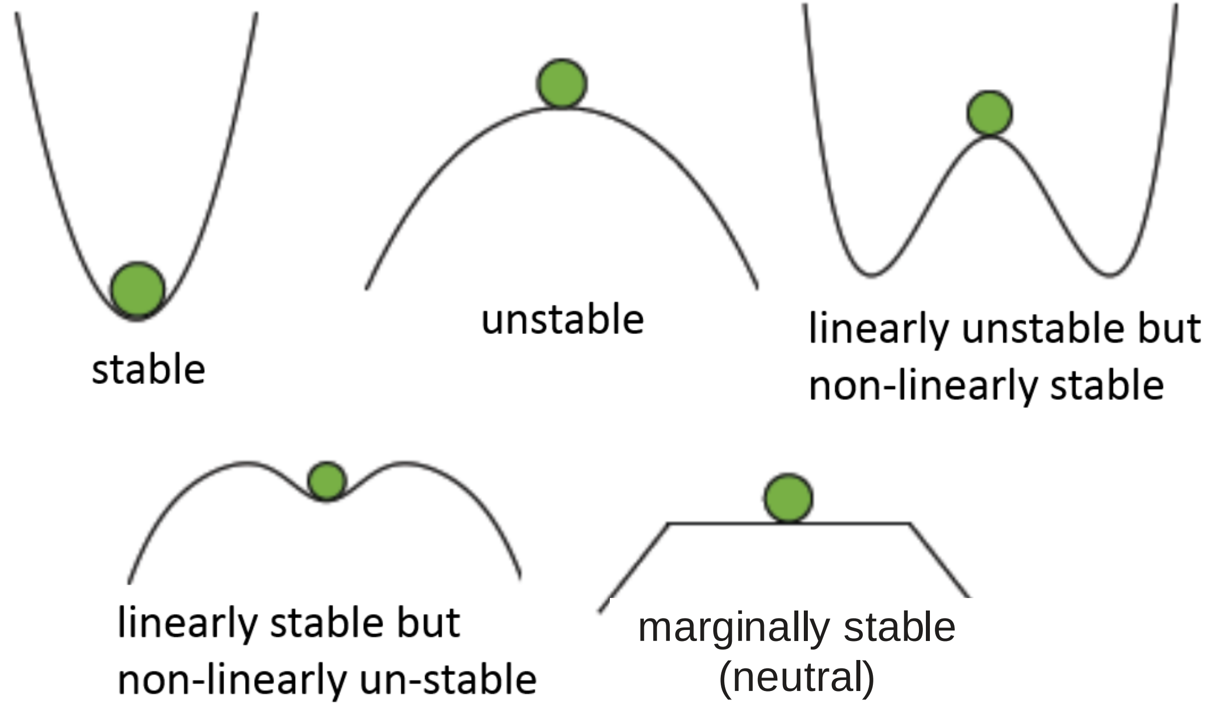
- ➢ Necessary, but not sufficient condition for plasma confinement

Def.: Stable MHD equilibrium ≡ Forces resulting from <u>any</u> small perturbation are directed to restore equilibrium

- May allow plasma confinement over longer time scales

- Important to understand whether the MHD equilibrium is stable to small perturbations
  - ➢ **Will the plasma configuration survive or ultimately collapse?**
  - ➢ **Will the plasma change its configuration?**

Analogy with classical mechanics



Conceptual example: Sausage instability of the Z-pinch

- Z-pinch with axial perturbation in B_θ ($k_z \neq 0$, $m=0$)

Complex notation

$$\frac{d}{dr}\left[p(r) + \frac{B_\theta^2(r)}{2\mu_0}\right] + \frac{B_\theta^2(r)}{\mu_0 r} = 0$$

Perturbation: $\propto \exp(ik_z z + im\theta)$

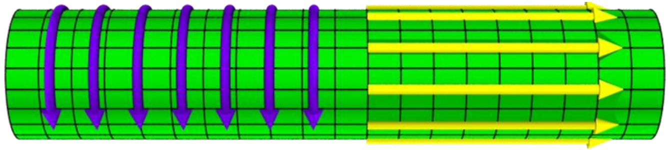


Image Credits: Wikipedia Creative Commons

Field $B_\theta(r)$

Current $j_z(r)$

Conceptual example: Sausage instability of the Z-pinch

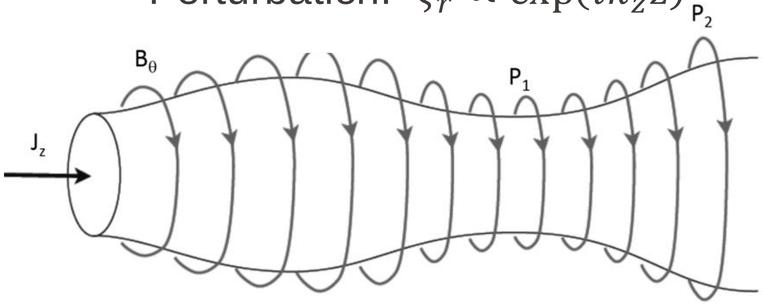
- Z-pinch with axial perturbation in B_θ ($k_z \neq 0$, $m=0$)

- **P_1**: B_θ is stronger than equilibrium
  - ➢ Magnetic pressure + field line tension > plasma pressure
  - ➢ Plasma is compressed in phase with the perturbation

- **P_2**: B_θ is weaker than equilibrium
  - ➢ Magnetic pressure + field line tension < plasma pressure
  - ➢ Plasma expands in phase with the perturbation

$$\frac{d}{dr}\left[p(r) + \frac{B_\theta^2(r)}{2\mu_0}\right] + \frac{B_\theta^2(r)}{\mu_0 r} = 0$$

Perturbation: $\xi_r \propto \exp(ik_z z)$



Reminder: $B_\theta(r) = \frac{\mu_0 I_z(r)}{2\pi r}$

- Net global effect: the plasma is compressed and rarified in phase with the perturbation ➔ **sausage instability**

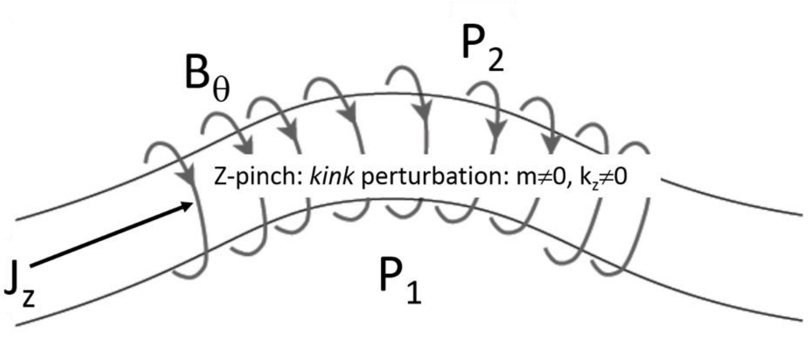
Conceptual example: Kink instability of the Z-pinch

- Z-pinch with azimuthal perturbation in B_θ ($k_z \neq 0$, $m=1$)

- Field lines are closer in the region P_1, and more distant in the region P_2
  - **P_1**: B_θ is stronger than the equilibrium value
  - **P_2**: B_θ is weaker than the equilibrium value

$$\frac{d}{dr}\left[p(r) + \frac{B_\theta^2(r)}{2\mu_0}\right] + \frac{B_\theta^2(r)}{\mu_0 r} = 0$$

Perturbation: $\xi_r \propto \exp(ik_z z + im\theta)$

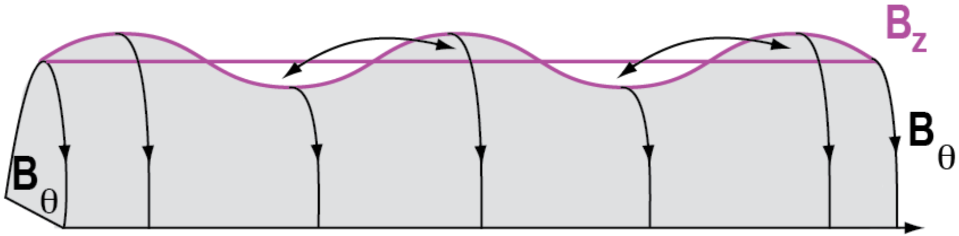


- Net global effect: the perturbed force is in phase with the perturbation

➔ **kink instability**

Conceptual example: Kink instability of the screw pinch

- Add an axial (toroidal) field B_Z ➔ screw pinch
- Displacement of the kink (or sausage) instability bends field lines ➔ stabilising effect

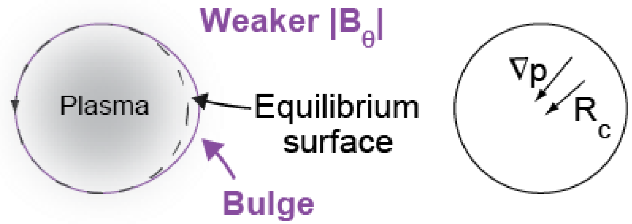


[Figure adapted from J. Freidberg, PP and FE]

➢ Axial (toroidal) field determines maximum axial (toroidal) current

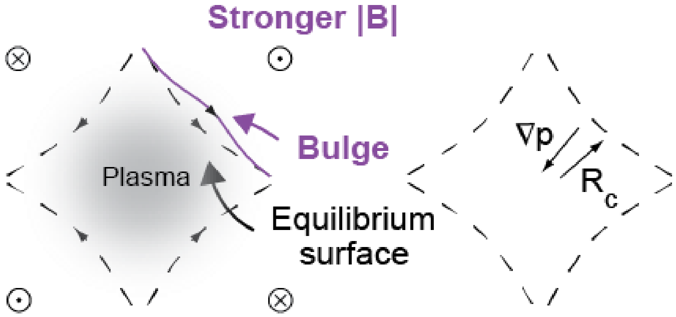
Interchange stability introduces the concept of good and bad curvature

- Curvature of field lines determine response to a ‘bulge’
  - Convex field lines are prone to interchange (e.g. Z-pinch)



- Field line curvature and pressure gradient in same directions = **‘bad curvature’**

  - Concave field lines resist interchange (e.g. magnetic cusp)

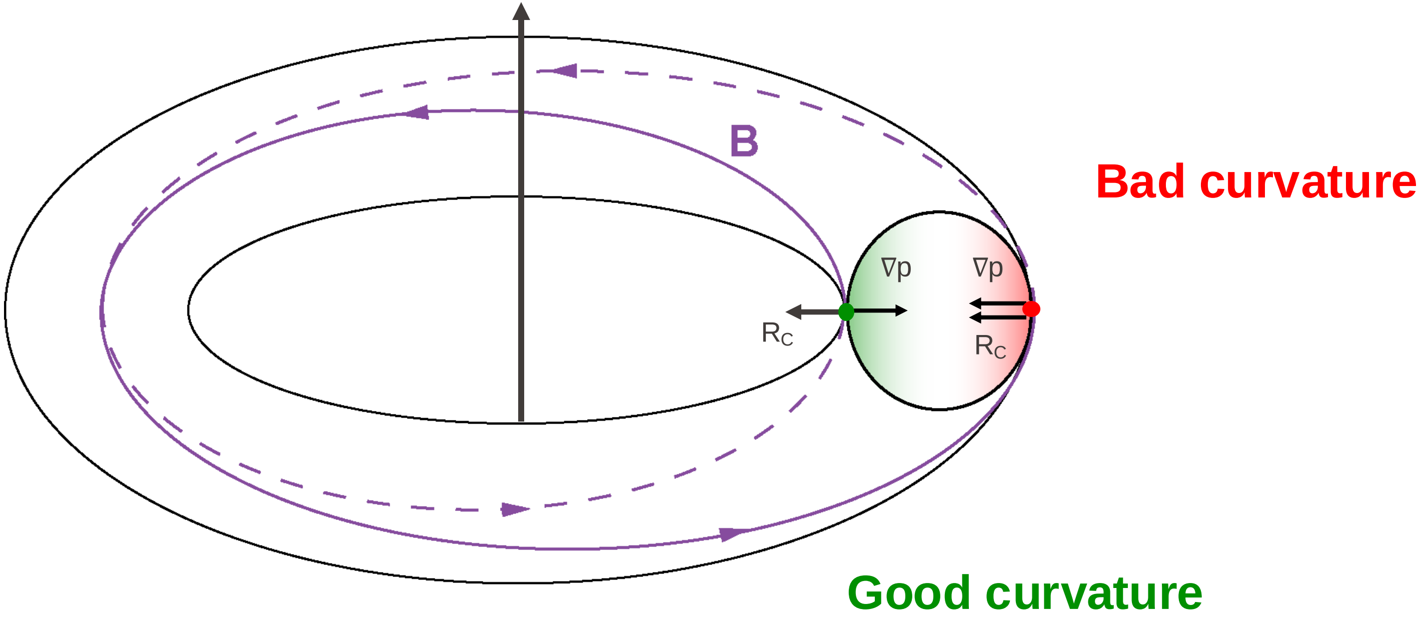


- Field line curvature and pressure gradient in opposite direction = **‘good curvature’**

[Figures adapted from J. Freidberg, PP and FE]

Toroidicity introduces regions of 'good' and 'bad' curvature

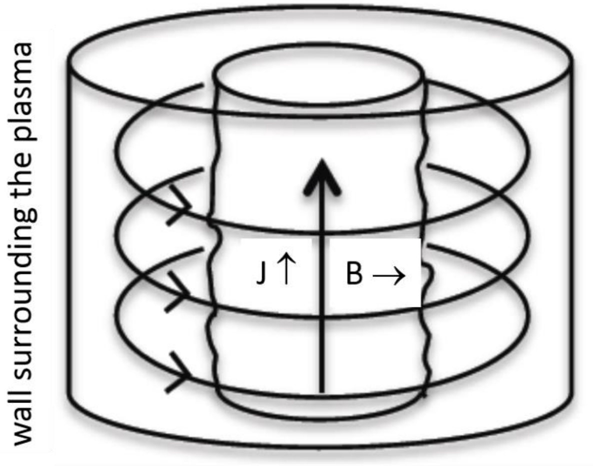
- In the presence of a strong toroidal field ('tokamak') toroidal curvature dominates the field line geometry



Wall effect on MHD instabilities

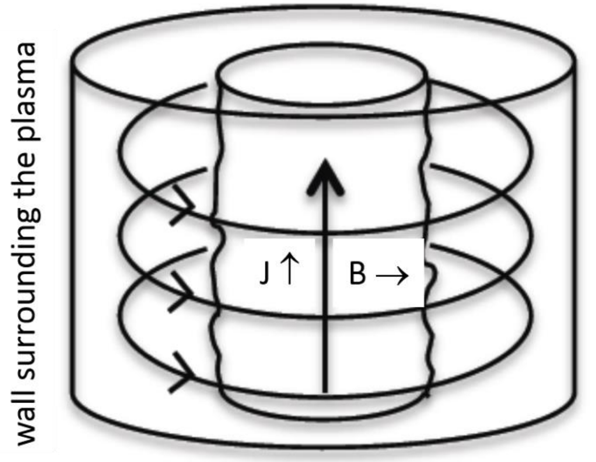
- Plasma with current $\bar{J}$ and magnetic field $\bar{B}$
- An instability develops that pushes the plasma towards a surrounding ideal wall ($\eta = 0$)
- The magnetic field cannot penetrate into the wall

- What happens, if the plasma is displaced towards the wall?



Wall effect on MHD instabilities

- As $d\Phi/dt = 0$ in ideal MHD (see L3), the magnetic flux through every surface moving with the plasma is constant
- A displacement of the plasma towards the wall compresses the flux surfaces in the vacuum region between the plasma and the wall
- The magnetic pressure is increased and pushes the plasma back



➢ Plasmas can be stabilized by a surrounding wall

- However: finite resistivity of the wall allows for flux diffusion through the wall and limits this effect to a finite time scale (typically of the order of milliseconds)

➔ Resistive Wall Modes (RWM)

General principles for stability analysis

- Fourier (normal mode) analysis of small perturbations

$$\propto \exp\left(i\bar{k}\bar{x} - i\omega t\right)$$

Complex notation

➢ Sign of Im(ω) determines stability ➔ Im(ω)>0 corresponds to instability

General principles for stability analysis

- Cast MHD equation into equation of motion

$$\rho_M \, \partial^2 \bar{\xi} / \partial t^2 = \bar{F}(\bar{\xi})$$

where ξ is a fluid displacement

- Fourier analysis in time ($\bar{\xi} \propto e^{-i\omega t}$) yields an eigenvalue equation

$$-\rho_M \omega^2 \bar{\xi} = \bar{F}(\bar{\xi})$$

➔ sign(ω^2)=+1/-1 corresponds to stability/instability

- Energy principle analysis: evaluate the change in potential energy $\delta W = -1/2 \int_V \bar{F}(\bar{\xi}) \cdot \bar{\xi} dV$ due to a displacement $\bar{\xi}$

➔ sign(δW)=+1/-1 corresponds to stability/instability

Linear stability analysis

- Linear stability analysis is a frequently used mathematical technique to evaluate the MHD stability of equilibria
  1. Linearise all fluid and MHD equations

$$Q(\bar{r},t) = Q_0(\bar{r}) + Q_1(\bar{r},t)$$

  - Q_0 : equilibrium value, i.e. $\partial Q_0/\partial t = 0$
  - $Q_1 \ll Q_0$: small perturbation to the equilibrium
  - $\varepsilon = |Q_1/Q_0|$: linear expansion parameter

  2. Taylor expand functions of perturbed parameters

$$F(Q) = F(Q_0 + Q_1) = F(Q_0) + \frac{\partial F(Q_0)}{\partial Q} Q_1 + \frac{1}{2}\frac{\partial^2 F(Q_0)}{\partial Q^2} Q_1^2 + \ldots.$$

  3. Use that equilibrium parameters ($Q_0, \ldots$) satisfy force balance
  4. Keep only terms that are of order ε

Apply linear stability analysis to ideal MHD equations

- Expand all dependent variables
  - $\bar{B} = \bar{B}_0 + \bar{B}_1, \quad \bar{J} = \bar{J}_0 + \bar{J}_1, \quad p = p_0 + p_1, \quad \rho = \rho_0 + \rho_1$
  - Static equilibrium: $\bar{v} = \bar{v}_1$
- Unperturbed variables satisfy equilibrium equations
  - Force balance: $\bar{J}_0 \times \bar{B}_0 - \nabla p_0 = 0$
  - Ampere's law: $\nabla \times \bar{B}_0 = \mu_0 \bar{J}_0$
  - Gauss's law: $\nabla \bar{B}_0 = 0$
- Linearise equations

Ex.: Linearise force balance equation

- Force balance: $\rho \frac{\partial \bar{v}}{\partial t} = \bar{J} \times \bar{B} - \nabla p$

Apply linear stability analysis to ideal MHD equations

- Expand all dependent variables
  - $\bar{B} = \bar{B}_0 + \bar{B}_1, \ \bar{J} = \bar{J}_0 + \bar{J}_1, \quad p = p_0 + p_1, \quad \rho = \rho_0 + \rho_1$
  - Static equilibrium: $\bar{v} = \bar{v}_1$
- Equilibrium equations
  - Force balance: $\bar{J}_0 \times \bar{B}_0 - \nabla p_0 = 0$
  - Ampere's law: $\nabla \times \bar{B}_0 = \mu_0 \bar{J}_0$
  - Gauss's law: $\nabla \bar{B}_0 = 0$
- Linearise equations, e.g. force balance

$$\rho_0 \frac{\partial \bar{v}_1}{\partial t} = \bar{J}_0 \times \bar{B}_1 + \bar{J}_1 \times \bar{B}_0 - \nabla p_1$$

- Assume same time dependence for all perturbed quantities

$Q_1 \propto exp(-i\omega t)$ (normal mode expansion)

Apply linear stability analysis to ideal MHD equations

- Introduce fluid displacement $\bar{\xi}(\bar{x},t) \quad \Rightarrow \quad \frac{\partial \bar{\xi}(\bar{x},t)}{\partial t} = \bar{v}_1(\bar{x},t)$

➢ Force balance equation

$$\rho_0 \frac{\partial^2 \bar{\xi}(\bar{x},t)}{\partial t^2} = \bar{F}\left(\bar{\xi}(\bar{x},t)\right) = -\omega^2 \rho_0 \bar{\xi}(\bar{x},t)$$

with a force operator

$$\bar{F}(\bar{\xi}) = \bar{J}_0 \times \bar{B}_1 + \bar{J}_1 \times \bar{B}_0 - \nabla p_1$$

$$= \frac{1}{\mu_0}(\nabla \times \bar{B}_0) \times \bar{B}_1 + \frac{1}{\mu_0}(\nabla \times \bar{B}_1) \times \bar{B}_0 - \nabla p_1$$

After using Ampère's law

Perturbed field and perturbed pressure depend on displacement

- Perturbed field $\bar{B}_1$: Combine Faraday and Ohm's law

$$\frac{\partial \bar{B}_1}{\partial t} = \nabla \times \bar{E}_1 = \nabla \times (\bar{v}_1 \times \bar{B}_0) \;\; \rightarrow \;\; \bar{B}_1 = \nabla \times \left(\bar{\xi} \times \bar{B}_0\right)$$

- Perturbed pressure p_1: Combine adiabatic equation of state and continuity

$$\frac{\partial p_1}{\partial t} = -p_0 \gamma \nabla \cdot \bar{v}_1 - \bar{v}_1 \cdot \nabla p_0 \;\; \rightarrow \;\; p_1 = -p_0 \gamma \nabla \cdot \bar{\xi} - \bar{\xi} \cdot \nabla p_0$$

Apply linear stability analysis to ideal MHD equations

- Introduce fluid displacement $\bar{\xi}(\bar{x},t)$ $\Rightarrow$ $\frac{\partial \bar{\xi}(\bar{x},t)}{\partial t} = \bar{v}_1(\bar{x},t)$

➢ Force balance equation

$$\rho_0 \frac{\partial^2 \bar{\xi}(\bar{x},t)}{\partial t^2} = \bar{F}\left(\bar{\xi}(\bar{x},t)\right) = -\omega^2 \rho_0 \bar{\xi}(\bar{x},t)$$

with a force operator (after using Ampère's law)

$$\bar{F}(\bar{\xi}) = \frac{1}{\mu_0}(\nabla \times \bar{B}_0) \times \bar{B}_1 + \frac{1}{\mu_0}(\nabla \times \bar{B}_1) \times \bar{B}_0 - \nabla p_1$$

- Ideal MHD force operator $\bar{F}(\bar{\xi})$ is self-adjoint, i.e. satisfies the property $\int \bar{\eta}^* \bar{F}(\bar{\xi}) d\bar{r} = \int \bar{\xi}^* \bar{F}(\bar{\eta}) d\bar{r}$ ➔ Eigenvalues ω^2 of $\bar{F}(\bar{\xi})$ are real

MHD equilibrium: linear stability analysis

- MHD energy principle: work done by moving the plasma through a fluid distance element ξ

$$\delta W = -\frac{1}{2}\int \bar{F}(\bar{\xi})\cdot\bar{\xi}\,dV$$

$$= \frac{1}{2}\int\left(\gamma p_0(\nabla\cdot\bar{\xi})^2 + (\bar{\xi}\cdot\nabla p_0)\nabla\cdot\bar{\xi} + \frac{\bar{B}_1^2}{\mu_0} + \bar{J}_0\cdot(\bar{\xi}\times\bar{B}_1)\right)dV + \frac{1}{2}\int\left(p_1 + \frac{\bar{B}_0\cdot\bar{B}_1}{\mu_0}\right)\bar{\xi}\cdot d\bar{S}$$

always>0 stabilising

depends ➔ pressure driven modes

always>0 stabilising

depends ➔ current driven modes

depends ➔ role of the wall enclosing the plasma

or $+\int_{\mathrm{vac}}\frac{B_v^2}{2\mu_0}\,dV$

➢ Sign of δW determines stability of the system

Example: Current-driven modes

- Circular, large aspect ratio, low- β tokamak
  - Large aspect ratio: $B_\phi, R \sim const.$ $\quad (\rightarrow q = rB_\phi/(RB_\theta))$
  - Low β: $p_0 \sim 0$ (consider only *current-driven* modes)

$$\delta W = \pi R \int_0^a \left(\frac{\bar{B}_1^2}{\mu_0} + j_{\phi 0}(B_{r1}\xi_\theta - B_{\theta 1}\xi_r) \right) d\theta r dr + 2\pi \int_a^b \frac{B_v^2}{2\mu_0} d\theta r dr$$

- Use normal mode test function $\xi \propto e^{i(m\theta - n\phi)}$

$$\delta W = \frac{\pi^2 B_\phi^2}{\mu_0 R} \left\{ \int_0^a \left[\left(r\frac{d\xi}{dr} \right)^2 + (m^2-1)\xi^2 \right] \left(\frac{n}{m} - \frac{1}{q} \right)^2 r dr + \left[\frac{2}{q_a}\left(\frac{n}{m} - \frac{1}{q_a} \right) + (1+m\lambda)\left(\frac{n}{m} - \frac{1}{q_a} \right)^2 \right] a^2 \xi_a^2 \right\}$$

Example: Current-driven modes (cont.)

- Inspect (potential) energy functional

$$\delta W = \frac{\pi^2 B_\phi^2}{\mu_0 R}\left\{\int_0^a \left[\left(r\frac{d\xi}{dr}\right)^2 + (m^2-1)\xi^2\right]\left(\frac{n}{m}-\frac{1}{q}\right)^2 r dr + \left[\frac{2}{q_a}\left(\frac{n}{m}-\frac{1}{q_a}\right) + (1+m\lambda)\left(\frac{n}{m}-\frac{1}{q_a}\right)^2\right] a^2\xi_a^2\right\}$$

- Plasma contribution least stable for $m = 1 \wedge \xi = const.$ when δW_{plasma}=0
  - Ideal wall at $r = a \;\rightarrow\; \xi_a = 0 \;\rightarrow$ need to go to higher order expansion of δW_{plasma} (**internal kink mode**)
  - $\xi_a \neq 0 \rightarrow \delta W_{\text{vacuum}}$ determines stability. Assume no wall ($\lambda = 1$): $q_a > \frac{1}{n}$ for stability

Example: Current-driven modes (cont.)

- Inspect (potential) energy functional

$$\delta W = \frac{\pi^2 B_\phi^2}{\mu_0 R}\left\{\int_0^a \left[\left(r\frac{d\xi}{dr}\right)^2 + (m^2-1)\xi^2\right]\left(\frac{n}{m}-\frac{1}{q}\right)^2 r dr + \left[\frac{2}{q_a}\left(\frac{n}{m}-\frac{1}{q_a}\right) + (1+m\lambda)\left(\frac{n}{m}-\frac{1}{q_a}\right)^2\right] a^2\xi_a^2\right\}$$

- All modes with $\frac{m}{n} < q_a$ stable for any wall position
  - For plasmas where q increases with r, current driven modes with resonant surface inside the plasma are stable

Stability of the MHD equilibrium: summary

- Main class of instabilities: sausage (interchange), kink, tearing
  - Wall surrounding the plasma can be stabilizing because of the frozen-in flux condition in ideal MHD
- Mathematical approach: linearise fluid and Maxwell's equations
  - Eigenvalue analysis to determine stability
  - Energy principle to determine stability

Outline

- MHD stability of the tokamak configuration
  - Conceptual examples of instabilities
  - Linear stability analysis
  - Waves in ideal MHD
- **Operational limits in tokamak plasmas**

Performance and operational limits of modern fusion devices

- Fusion performance (see L1)
  - To produce thermonuclear fusion in magnetically confined plasmas we need $n\tau_E \sim 10^{20} m^{-3}s$ for $T \geq 10keV$
  - For thermonuclear fusion to be economically attractive we need an engineering fusion energy gain in the range $2 \leq Q_E \leq 10$ (see L1 notes)

⇩

  - This corresponds to a physics fusion energy gain in the range $10 \leq Q \leq 40$
- Operational limits: What limits the attainable Q?

Fusion performance and operational limits

- Break-even: a sufficient $nT\tau_E$ has to be reached at $T \geq 10keV$
- Energy confinement time increases with current (see L5) ➔ $\tau_E \propto I_P$
- Fusion power increases with plasma pressure ➔ $P_{fus} \propto (nT)^2 \propto \beta^2$ (for 5keV < T < 15keV)
- I_p, n, and p are limited by different mechanisms: operational limits
- Approaching operational limits leads to disruptions (hard limit), or confinement degradation (soft limit)
- How to build an economically viable fusion reactor?
  - Fix B_ϕ as high as possible ➔ then maximise $\{I_P, n, \beta\}$ to minimise V (as cost $\propto$ size)

➔ The maximum values of $\{I_P, n, \beta\}$ are all limited by MHD instabilities!

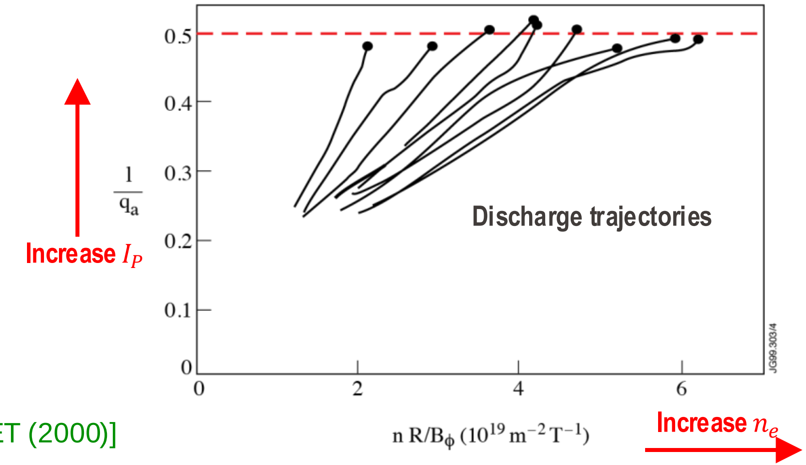
H. Reimerdes

Operational limit: plasma current

- Toroidal field has to be sufficiently large to suppress the kink instability driven by the poloidal field
  - Ratio of toroidal and poloidal field expressed by the safety factor

$$q_a = \frac{aB_{\phi,0}}{R_0 B_{\theta,a}} = \frac{2\pi a^2 B_{\phi,0}}{\mu_0 R_0 I_P} \quad \text{(for a circular cylindrical plasma)}$$

- Stability requires $q_a \geq 2$



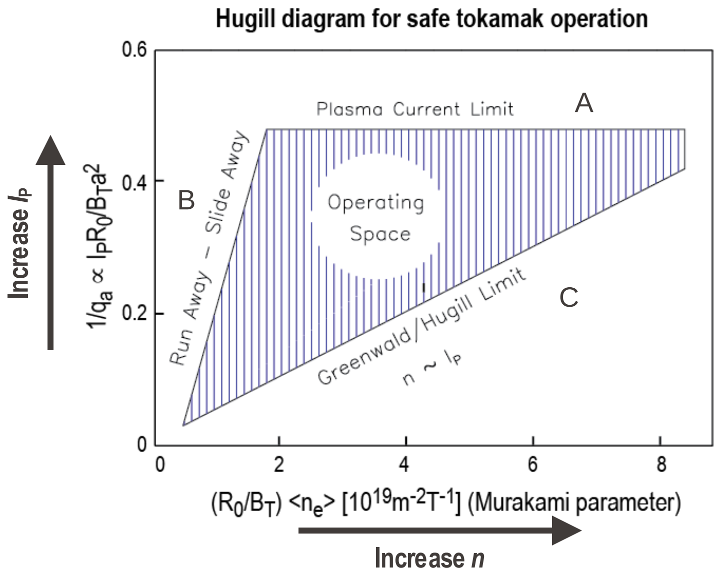
[Figure from J. Wesson, The Science of JET (2000)]

Operational limits: Hugill's diagram

- Hugill's diagram for safe operation
  - **A**: Minimum safety factor q_a>2 required to avoid current-driven kink instability
  - **B**: Minimum density required to avoid generation of runaway electrons
  - **C**: Maximum density increases with plasma current (Greenwald limit)

- Operation beyond limits of the Hugill's diagram

  ➔ disruptions



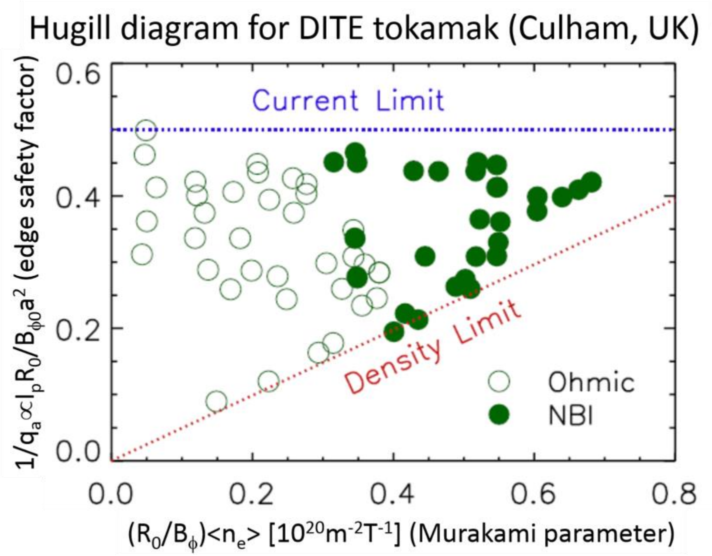
[Figure adopted from M. Greenwald, PPCF (2002)]

Operational limits: plasma density

- Greenwald density limit: the maximum achievable density depends on plasma current and plasma size

  ➔ $n \leq n_G = I_P/(\pi a^2)$

  - With n_G in 10^{20}m^{-3}, I_p in MA and a in m
  - Exceeding the Greenwald density limit typically leads to disruption



Operational limits: (normalized) plasma pressure

- Troyon limit: limit of normalized plasma pressure β due to global ideal MHD kink mode scales as $\beta_{\max} = C_\beta \; I_\mathrm{P}/(aB_0)$
  - C_β~2→5 when optimizing plasma shaping

- Definition of normalized beta $\beta_\mathrm{N} \equiv \dfrac{\beta}{I_\mathrm{P}/(aB_0)}$
  - With β in %, *a* in m, *B* in T and I_p in MA

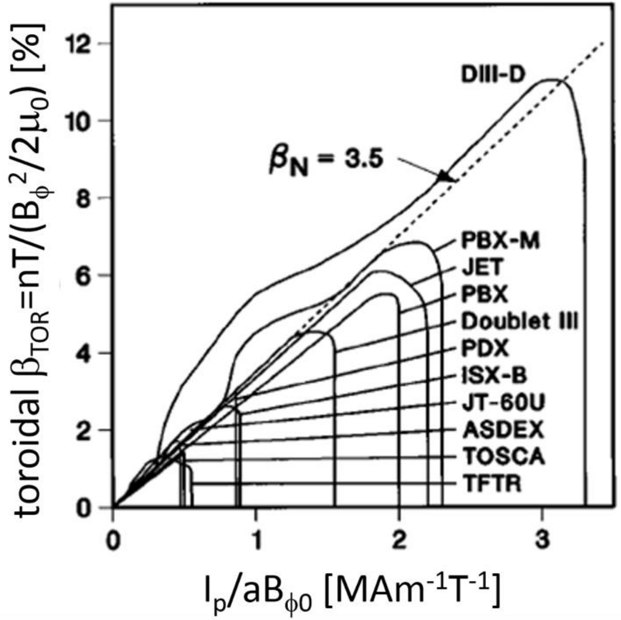
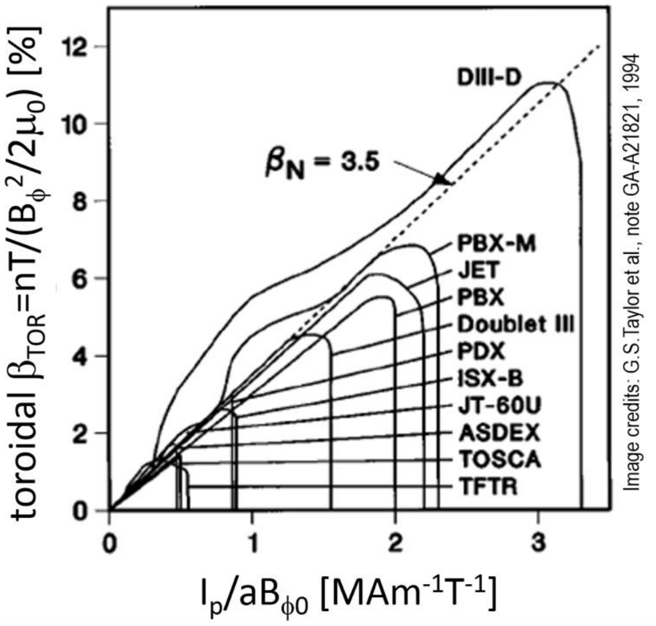


Image credits: G.S.Taylor et al., note GA-A21821, 1994

Operational limits: (normalized) plasma pressure

- Troyon limit: limit of normalized plasma pressure β due to global ideal MHD kink mode scales as $\beta_{\max} = C_\beta \; I_P/(aB_0)$
  - C_β~2→5 when optimizing plasma shaping
- Definition of normalized beta $\beta_N \equiv \frac{\beta}{I_P/(aB_0)}$
  - With β in %, *a* in m, *B* in T and I_p in MA
- In practice β-limit usually set by resistive MHD instabilities in the vicinity of ideal MHD limit

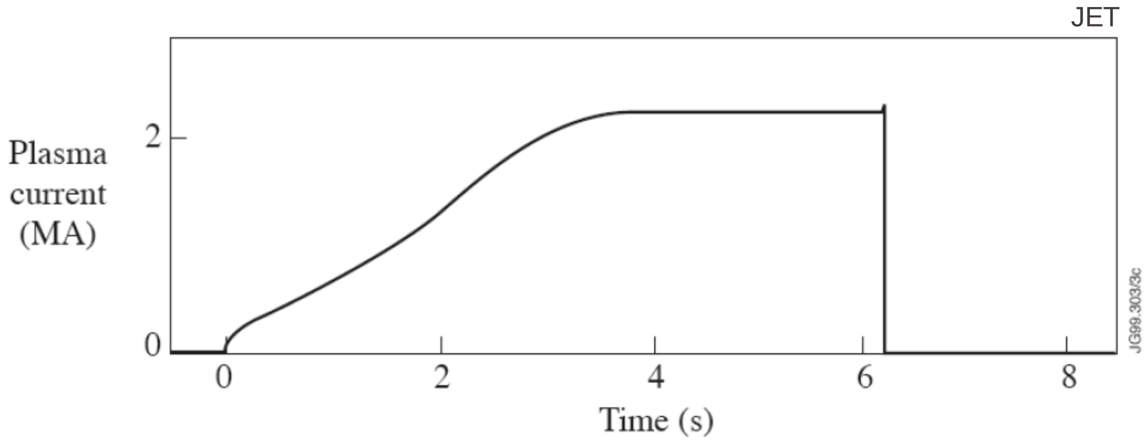


Operational limits: summary

- Current limit: plasma current limited by minimum safety factor $q_a \geq 2$ to avoid current driven kink mode
  - Plasma current I_p exceeds safety factor limit ➔ disruption
- Greenwald limit: maximum achievable density $n_{\max} \leq n_{\mathrm{G}}$ with Greenwald density $n_{\mathrm{G}} = I_{\mathrm{P}}/(\pi a^2)\ [10^{20}/m^3, MA, m]$
  - Exceeding the Greenwald limit ➔ disruption
- Troyon limit: maximum achievable β scales as $\beta_{\max} = C_\beta\ I_{\mathrm{P}}/(aB_0)\ [\%, \mathrm{MA}, \mathrm{m}, \mathrm{T}]$
  - C_β~2➔5 when optimizing plasma shaping
  - Exceeding the Troyon-limit ➔ disruption
- Disruption: an exceptionally rapid I_p quench in tokamaks
  - Plasma energy fully lost in ~1ms, several GWs dumped onto reactor wall ➔ serious damage!

Operational limits: disruption

- Disruptions: exceptionally rapid loss of the plasma in tokamaks
  - Note: stellarators in principle disruptions-free
- Plasma energy lost in ~1ms and several GWs (e.g. in JET) dumped onto device's wall

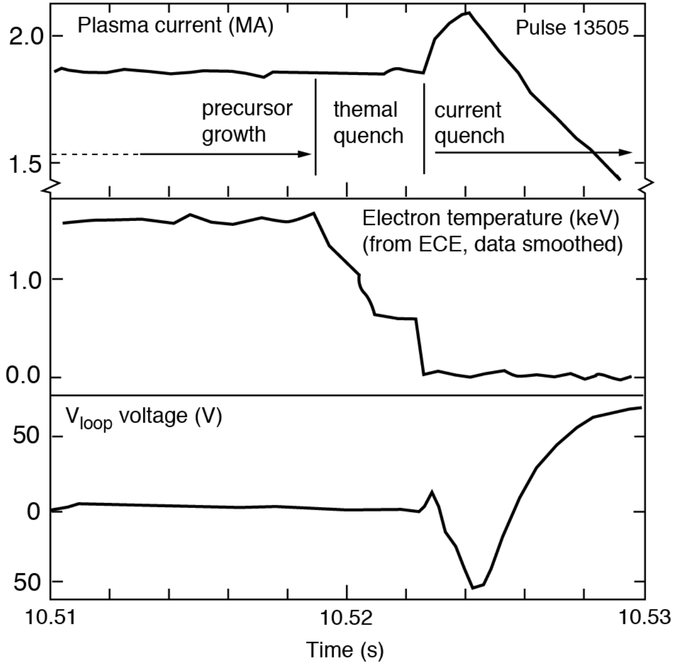


[Figure from J. Wesson, The Science of JET (2000)]

Operational limits: disruption

- Phase 1: Precursor growth
  - Instability growth may already degrade confinement (typical τ~10ms)
- Phase 2: Thermal quench
  - Rapid loss of the kinetic plasma energy (typical τ~1ms) to limiting surfaces (~10MJ in JET)
  - Cooling flattens current profile and induces a current spike
- Phase 3: Current quench
  - Magnetic energy is dissipated through impurity radiation and eddy currents in the vessel wall

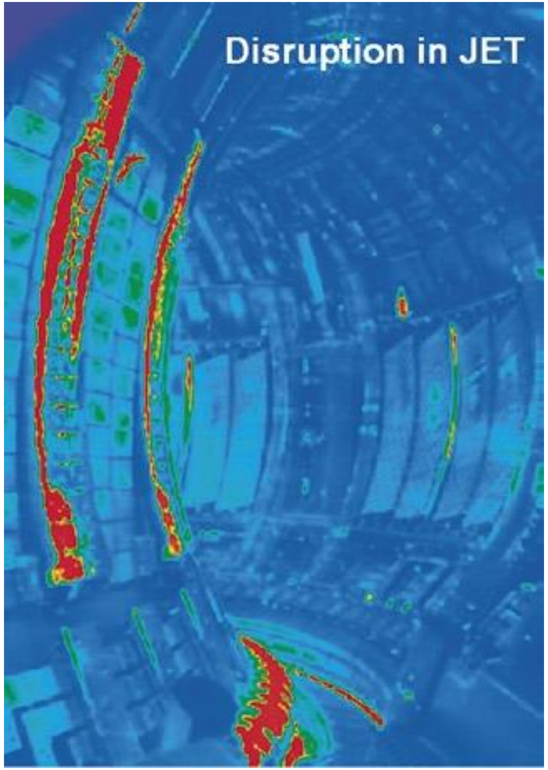
Locked mode disruption in the JET tokamak



[ITER Physics Basis, Chapter 3 (1999), Fig. 52]

Operational limits and MHD instabilities: disruption

- Disruptions: may cause damage to plasma facing components
- Melting: metals melt and carbons sublimate when the heat flux exceeds the limits of the materials
- ITER: only ~5 *very minor* disruptions allowed over its entire ~30yrs life-time!



Fusion performance, operational limits: summary

- Fusion performance: scales as $P_{\mathrm{fus}} \propto \beta^2$ for the typical operating range of {density, temperature}
  - I_p, n, and p are of outmost importance for optimizing a reactor
  - These parameters are constrained within limited operational range described by theoretical and empirical scaling laws
- Operational limits: affect maximum $\{n, \beta, I_P\}$ values that can be achieved in the optimal temperature range for fusion
- Approaching operational limits may lead to disruptions (hard limit) or confinement degradation (soft limit)
- Passive control of instabilities: use intrinsic stabilization mechanisms, e.g. by the wall surrounding the plasma
- *Active control: detect the onset of an instability, and apply feedback control schemes in real-time to stabilize the instability or limit its development*

Additional material

Classification of instabilities

- Various classification schemes exist
- Internal and external modes
  - Does the plasma surface (have to) move as the instability grows?
    - Only external modes can benefit from wall stabilisation (distinguish no-wall and conducting wall modes)
    - Internal modes typically do not lead to catastrophic loss of plasma
- Pressure-driven and current-driven modes
  - Pressure driven modes include modes driven by perpendicular current (combination of pressure gradient and curvature radius)
  - Pressure-driven modes may be ‘interchange’ or ‘ballooning’
  - Current driven modes may even exist at low beta and are also called “kink-modes”

Safety factor – large aspect ratio, elliptical cross section

- Safety factor: Average number of toroidal turns per poloidal turn of a field line

$$q(r) = \frac{rB_{\phi,0}}{R_0 B_\theta(r)} \sqrt{\frac{1+\kappa^2}{2}}$$

  - Link flux surface averaged B_θ to enclosed current $I(r)$

$$2\pi r \sqrt{\frac{1+\kappa^2}{2}} B_\theta(r) = \mu_0 I(r)$$

  - Dependence of safety factor on plasma current

$$q(r) = \frac{2\pi r^2 B_{\phi,0}}{\mu_0 R_0 I(r)} \frac{1+\kappa^2}{2}$$